Introduction to Caffeine

[1]

In this lesson, we explore the dynamics of caffeine in the body through the use of exponential functions. Various foods and drinks popular around the world contain caffeine. Caffeine is an alkaloid compound that comes from plants, including coffee, tea, kola nuts, mate, cacao and guarana. Many people drink caffeine drinks because they like the taste of them, others for the physical effect of the caffeine. Most people are aware of differences in the way they feel as a result of drinking caffeine, which stimulates the central nervous system, the heart muscles, and the respiratory system. The way individuals interpret the effects of caffeine as a stimulant varies widely. For many, the effect is pleasant and energizing, a "wake up" or a "pick-me-up", and it can delay fatigue. For others, the effects are unpleasant. Laboratory tests indicate that 1 to 3 cups of coffee can produce an increased capacity for sustained intellectual effort and decrease reaction time, but may adversely affect tasks involving delicate muscular coordination and accurate timing.

The effect of coffee is quite different from the effect of alcohol, for example, with regard to increase in mental capacity; such an increase is not seen in persons intoxicated with alcohol. Caffeine is classified as a stimulant, whereas alcohol is a depressant. These two general classes of drugs have very different effects on the human body.

Quantity of caffeine per drink The amount of caffeine in different drinks varies, and some also contain other alkaloids that act as stimulants or relaxants. Thus, it is difficult to relate the amount of caffeine in a drink to the physical effect it may have on your body. We list an average, approximate amount of caffeine in some drinks. The caffeine levels in commercial sodas tend to be consistent. The caffeine levels in coffee and tea vary widely according both to the plant and to processing, but these numbers give some idea of the caffeine level.

Drip	165 mg
Brewed	130 mg
Instant	95 mg
Decaffeinated	4 mg

Table 1: Caffeine in 8oz cup of coffee.

Brewed	45 mg
Instant	35 mg
Green tea	30 mg

Table 2: Caffeine in 8oz cup of tea.

[1] material originally written by Rosalie Dance and James Sandefur

Coca-Cola	45.6 mg
Diet coke	45.6 mg
Surge	51 mg
Dr. Pepper	39.6 mg
Pepsi	37.2 mg
Diet Pepsi	35.4 mg
Mountain Dew	55 mg

Table 3: Caffeine in 12oz can of Soda.

The effects of caffeine can only be felt when the caffeine is present in sufficient amounts. For most people, from 32 to 200 mg of caffeine acts as a minor stimulant; these amounts have been shown to speed up reactions in simple routinized tasks in laboratory experiments. Steadiness of the hand has been shown to be worse after 200 mg of caffeine. More than 300 mg is enough to produce temporary insomnia and 480 mg has been known to cause panic attacks in panic disorder patients. Amounts of 5 to 10 g (5000-10,000 mg) of caffeine cause death.

The two primary ways that chemicals are eliminated from the body are through filtration by the kidneys and metabolism by enzymes from the liver. Our bodies eliminate caffeine primarily by the functioning of the kidneys. The kidneys tend to filter out a constant proportion of a chemical, that proportion depending on the particular chemical and individual. In the "average person", about 13% of the caffeine in the body is eliminated each hour.

1. A person starts the day by drinking 3 cups of coffee containing 130 mg of caffeine each. How much caffeine will there be in this person's body 1 hour later? 2 hours later? 3 hours later?

2. Reflect on how you computed the answers to part a) to help you develop a relatively simple expression that gives the amount of caffeine in this person's body after 24 hours, assuming no additional caffeine is consumed. What is this amount? (You do not need to keep computing the amount of caffeine in the body for one hour after another until you reach 24 hours. You can compute this amount directly.)

3. Explain why
$$c(t) = 390\,(0.87)^t$$
gives the correct amount of caffeine in this person's body t hours after drinking the coffee containing 390 mg of caffeine.

4. Graph the function given in the previous part.

5. How long will it take until the caffeine is reduced to an amount that should no longer have any stimulant effect on the body?

6. How much caffeine will be in this person's body in 30 minutes (when $t = 1/2$)?

7. The time it takes for the amount of a substance present to be reduced by half is called the **half-life** of the substance. This term is applied in many situations, including the elimination of drugs from the body and the decay of radioactive materials. To the nearest tenth of an hour, how long will it take until the amount of caffeine in this person's body is cut in half; that is, from 390 to 195? How long will it take for the amount to be cut in half again, that is, to 97.5?

8. Find a value b, to two decimal places, such that the function

$$h(t) = 390\,(0.5)^{bt}$$

gives the correct amount of caffeine in this person's body after t hours. How does this function relate to the function $c(t)$ given earlier? How does the number b relate to the half-life found in the last part? Why does this make sense?

9. Find a value a such that the function

$$f(t) = 390\,e^{at}$$

gives the correct amount of caffeine in this person's body after t hours. How does this function relate to the function $c(t)$ given earlier?

10. A person gets up in the morning with no caffeine in her body. The following table gives the times at which she drank caffeinated drinks and the amount of caffeine in those drinks. Let $t = 0$ represent 8:00 am. Assume consumption and absorption of caffeine is immediate even though it may actually take some period of time consuming the drink and it may take a short period of time for the caffeine to be absorbed into your body. This means that at time $t = 0$, the amount of caffeine in her body is 250 mg, that is, $c(0) = 250$. Use this table to help sketch a graph of the amount of caffeine in this person's body for $0 \leq t \leq 10$. Use your graph to help write a conditionally defined function that gives the amount of caffeine in her body for $0 \leq t \leq 10$.

Time of caffeine consumption	8:00am	1:00 pm
Amount of caffeine consumption	250 mg	100 mg

Table 4: Caffeine consumption for person.