

Calculus, Math-035, Georgetown University
Limit of Slope of Secant Line

1) Suppose an object travels

$$f(t) = t^2$$

meters in t seconds.

a) What is the monotonicity of f ?

b) What is the average speed of the object during the interval of time $2 \leq t \leq 4$?

c) Draw the line connecting the two points on the curve, $(2,4)$ and $(4,16)$? What is the slope of that line?

d) What do the answers of the questions in problems 1 and 2 have in common? Explain.

2) Suppose that the function

$$R(x) = -0.015x^2 + 45x$$

gives the revenue in dollars a company earns for selling x items.

a) What is the revenue the company earns for selling 100 items?

b) What is the average revenue (dollars per item) that the company earns for selling the first 100 items?

c) What is the average revenue the company earns for selling the second 100 items?

d) Draw the line connecting the two points on the curve, $(100, R(100))$ and $(200, R(200))$. What is the slope of this line.

3) A grande cup of Starbucks coffee has about 260 mg of caffeine. The function that gives the mg of caffeine in a person's body t hours after drinking a grande coffee is

$$c(t) = 260(0.87)^t$$

a) Find the amount of caffeine in the body 2 hours after drinking a grande coffee.

b) What is the average rate of elimination of caffeine from the body during the first two hours, in mg per hour?

c) What is the average rate of elimination of caffeine from the body over the second two hours after drinking a grande coffee?

d) Graph the function c . Draw a line connecting $(0, c(0))$ and $(2, c(2))$ and find its slope.

e) Draw a line connecting $(2, c(2))$ and $(4, c(4))$ and find its slope.

4) Write a paragraph describing the relationship between finding the average change of a function $f(x)$ over an interval $[a, b]$ and the slope of the secant line connecting $(a, f(a))$ and $(b, f(b))$. This should include an explanation of what slope tells you and why it is important.