Calculus, Math-035, Georgetown University

Conditionally defined functions

Use the spreadsheet called

1. Consider the function

$$f(x) = \begin{cases} -x+3 & if \quad x \geq 1 \\ 3x+1.5 & if \quad x < 1 \end{cases}$$

graphed over the interval [-4, 6].

a. Use the formula to find $f(-1)$, $f(1)$, and $f(2)$.
b. Change the values of A, B, c, a, and b in the spreadsheet "Piecewise defined function" and use the spreadsheet to find $f(-1)$, $f(1)$, and $f(2)$.
c. Move the slider so that x_0 is about 0.5. Then click on the arrow at the right end of the slider repeatedly to increase the value of x_0. Describe what happens to the corresponding y value, especially as x_0 increases from less than 1 to exactly 1, and then to greater than 1.
d. Change only the value of B so that the graph is no longer disconnected at $x = 1$. What must the value of B equal? Why is this?

2. Enter the following values into the spreadsheet: xmin = 0, xmax = 5, A = -2, B = 8, c = 3, a = -0.5, and b = 2.

a. Write the piecewise definition (formula) for the resulting function.

b. On what interval(s), if any, is the function increasing?

c. On what interval(s), if any, is the function decreasing?

d. True or false: If a function is decreasing on an open interval (a, b) and also decreasing on (b, c), then the function is decreasing on the entire interval (a, c). Explain.

e. True or false: If a function is decreasing on a closed interval [a, b] and also decreasing on [b, c], then the function is decreasing on the entire interval [a, c]. Explain.

3. Draw a graph of a function that is decreasing on [0, 3] and increasing on [3, 5], but is one-to-one. Can you do this if we require that the graph be continuous? Explain.