Exercise sheet 3.7, 3.9 and 3.10

Exercise 1. Give the domain of differentiablity of the following functions f and their derivatives. Give the equation of the tangent to f at point 1 (when possible).

$f(x) = e^{\cos(x)}$

$f(x) = \tan(3x + 2)$

$f(x) = (\sqrt{x})^n$ with $n \in \mathbb{N}$

$f(x) = |x^2 - 1|$

$f(x) = e^{\lambda x}$ with $\lambda \in \mathbb{R}$

$f(x) = \sin(\cos(x))$

$f(x) = \sqrt{3x + 4}$

$f(x) = \sqrt{x^2 - 2}$

$f(x) = e^{3x^2 - 5x - 2}$

$f(x) = (\cos(x))^n$

Exercise 2. Give the domain of differentiability and the derivative of the function f^{-1} for the following functions f:

1. $f(x) = x$

2. $f(x) = 3x + 1$

3. $f(x) = e^x$

Exercise 3. Section 3.10. Theorem 1

1. What is the value of $\sin^2(x) + \cos^2(x)$ for all x?

2. Prove that $\sin(x) = \sqrt{1 - (\cos x)^2}$ and that $\cos(x) = \sqrt{1 - (\sin x)^2}$ for all x

3. Prove that $\sin^{-1}$ is differentiable on $(-1, 1)$ and that $(\sin^{-1})'(y) = \frac{1}{\sqrt{1-y^2}}$

4. Prove that $\cos^{-1}$ is differentiable on $(-1, 1)$ and that $(\cos^{-1})'(y) = -\frac{1}{\sqrt{1-y^2}}$

5. Prove that $\tan'(x) = 1 + (\tan x)^2$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

6. Prove that $\tan^{-1}$ is differentiable on $\mathbb{R}$ and that $(\tan^{-1})'(y) = \frac{1}{1+y^2}$