Exercise 1

True or False ? Tell if the following propositions are true or false. If you say "false", find a counter example. From now on, we assume that f is defined on $[-1, 1]$. We say that f is bounded if there exists l and L such that $l \leq f(x) \leq L$ for all x.

1. If f is bounded, then f has a maximum.
2. If f is differentiable and bounded, then f has a maximum.
3. If $f'(x) = 0$, then $f(x)$ is a local extremum of f.
4. If $f'(x) > 0$, and $x \in (-1, 1)$ then $f(x)$ can not be a local extremum of f.
5. If $f'(x) < 0$, then f can not be a local extremum of f.

Exercise 2

Find the critical points of f, that is, the points where $f'(c) = 0$ or does not exist. Determine the absolute extreme values of f or state that they don't exists.

1. $f(x) = x^2 - 10$ on $[-2, 3]$
2. $f(x) = \cos^2(x)$ on $[0, \pi]$
3. $f(x) = \sin(3x)$ on $\left[-\frac{\pi}{4}, \frac{\pi}{3}\right]$
4. $f(x) = (2x)^x$ on $[0.1, 1]$
5. $f(x) = x^2 + \cos^{-1}(x)$ on $[-1, 1]$
6. $f(x) = 2x^3 - 15x^2 + 24x$ on $[0, 5]$
7. $f(x) = \frac{4x^3}{3} + 5x^2 - 6x$ on $[-4, 1]$

Exercise 3

A sales analyst determines that the revenue from sales of fruit smoothies is given by $R(x) = -60x^2 + 300x$ where x is the price in dollars charged per items, for $0 \leq x \leq 5$.

1. Find the critical points of the revenue function.
2. Determine the absolute maximum value of the revenue function and the price that maximizes the revenue.