Exercise Sheet 4.2, part I

Increasing and decreasing functions

1) Using the definition, find the monotonicity of the function $f(x) = (x-3)^2 - 4$

2) Compute the derivative of f.

3) Use the following table to write the sign table of f'. We also represent the so-called "variation table" of f. What is the link between the sign table of f' and the variation table of f ?

x	$-\infty$		3		$+\infty$
$f'(x)$					
$f(x)$	$+\infty$	$\searrow$	-4	$\nearrow$	$+\infty$

Theorem 1

Suppose the f is continuous on an interval I and that it is differentiable at all interior point of I. If $f'(x) > 0$ on I, then f is increasing on I. If $f'(x) < 0$, then f is decreasing on I.

4) Try to use the definition to find the monotonically of the function $g(x) = \frac{x^3}{3} - x$.

5) Compute the derivative of g.

6) Use the theorem to draw the sign table of g' and the variation table of g.

x	$-\infty$ $+\infty$
$f'(x)$	
$f(x)$	

7) How many solutions are there to the equation $g(x) = 0$?