Homework 5
Due: 10/04

Exercise 1. Mid-term like exercise. Take 50 minutes to solve this exercise and write your answers in the blanks. When the time is off, you may continue to solve the exercise (and possibly correct some of your answers from this sheet) on a seperate sheet. Always justify your answers.

1. Let $f(x) = \begin{cases} \frac{x-3}{x^2-2x-3} & \text{if } x \neq 3 \\ \frac{1}{4} & \text{if } x = 3 \end{cases}$.

a) What is the domain of f?

b) What is the domain of continuity of f?

c) What are the asymptotes of f?

2) Find the following limits or state that they don't exist. Show your work.

a) $\lim_{x \to 2^+} \frac{3x^4+2x^2+1}{x^4-2x^3}$

b) $\lim_{x \to +\infty} \frac{\cos(x)e^{-x}}{2x^3+4x-2}$

c) $\lim_{x \to 2} \frac{2 + \frac{x}{x-3}}{x-2}$

d) $\lim_{x \to -\infty} \frac{\sqrt{-x^3 - 2x}}{-x+1}$

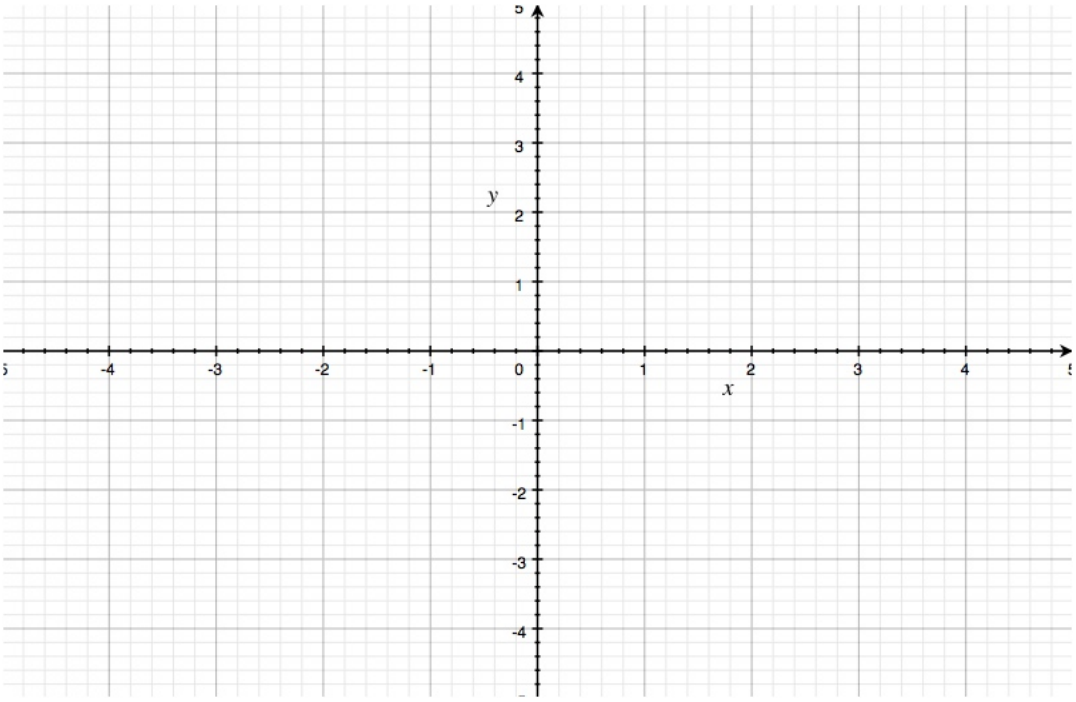
e) $\lim_{x \to +\infty} \sin(x)$

f) $\lim_{x \to 3^+} \frac{x-3}{\sqrt{x+1}-2}$

3) Let $h(x) = \begin{cases} \frac{x^2+3}{2x+4} & \text{if } x > -2 \\ \frac{x^2+3}{2x^2+4} & \text{if } x \leq -2 \end{cases}$

a) On what intervals is f continuous?

b) On the following grid, draw the asymptotes of f.



c) Using the asymptotes, draw how the function f looks like.

4) Let $g(x) = x^3 - x^2 - 2x + 1$.

a) What is the domain of continuity of g? Justify.

b) Compute $g(-2)$, $g(-1)$, $g(1)$, $g(2)$ and $\lim_{x \to +\infty} g(x)$.

c) Using the Intermediate value Theorem, tell how many solutions to the equation $g(x) = 0$ there are (at least).

Exercise 2. (this is not in the mid-term practice). Let $f(x) = \frac{3x-12}{x^2-3x-4}$.

a) What is the domain of f?

b) Compute $\lim_{x \to -1} f(x)$ or state that is does not exist.

c) Compute $\lim_{x \to 4} f(x)$ or state that it does not exist.

d) Is f continuous at 4 ?

e) We call a point y a "vanishing point" of g if $g(y)$ is not defined but $\lim_{x \to y} g(x)$ exists and is finite. Give an example of a function h that satisfies the following properties:

1. The point 1 is a vanishing point of h.
2. The line $y = 3$ is a vertical asymptote of h.
3. The line $x = 2$ is a horizontal asymptote of h.