

Homework 6

Due : 10/11

Exercise 1 Proofs from the lecture

1) Let f and g be two differentiable functions at point a . Prove that $f + g$ is differentiable at a and show that $(f + g)'(a) = f'(a) + g'(a)$.

2) Let f be a differentiable function at point a and $c \in \mathbb{R}$. Prove that $c \times f$ is differentiable at a and show that $(c \times f)'(a) = c \times f'(a)$.

Let $n \in \mathbb{N}$. The aim of the following questions is to prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ for all x .

3a) Prove that $a^n - b^n = (a - b) \times (a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + a^3b^{n-4} + a^2b^{n-3} + ab^{n-2} + b^{n-1})$ for all a and b .

3b) Use the last question to factorize $(x + h)^n - x^n$.

3c) Compute $\lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h}$ and conclude.

Exercise 2

Give the domain of differentiability of these functions and compute their derivative.

1. $f_1(x) = (x^2 + 1)e^x$

2. $f_2(x) = \frac{1}{2x-7}$

3. $f_3(x) = |x|$

4. $f_4(x) = \sqrt{x}(3x^8 + 5x^3 - 7x)$

5. $f_5(x) = \frac{1}{2\sqrt{x}}$

6. $f_6(x) = e^x \cos(x)$

7. $f_7(x) = \frac{3x^2 - x + 2}{x}$