Exercise sheet 4.2, part III

Let $f(x) = 3x^2 + x - 1$ and $g(x) = -2x^2 + 4x + 3$.

1) Compute $f'(-3)$, $f'(-2)$, $f'(-1)$, $f'(0)$, $f'(1)$, $f'(2)$ and $f'(3)$.

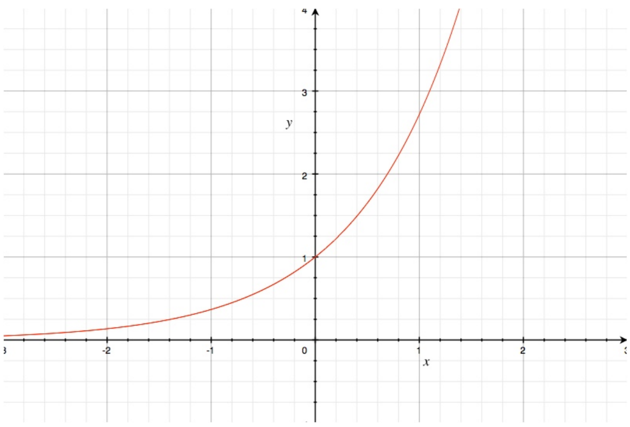
2) What can you tell about the montonicity of f'?

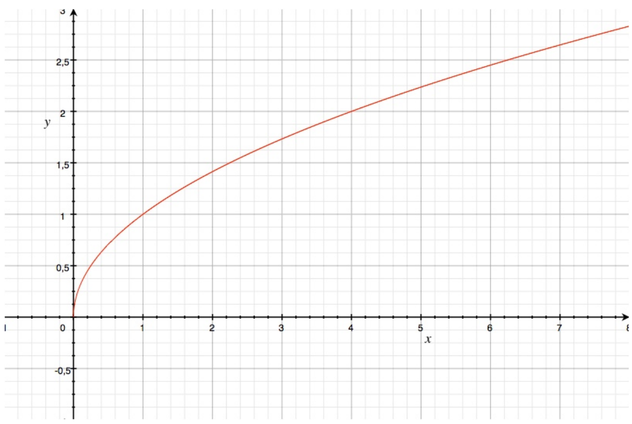
3) Compute $g'(-3)$, $g'(-2)$, $g'(-1)$, $g'(0)$, $g'(1)$, $g'(2)$ and $g'(3)$.

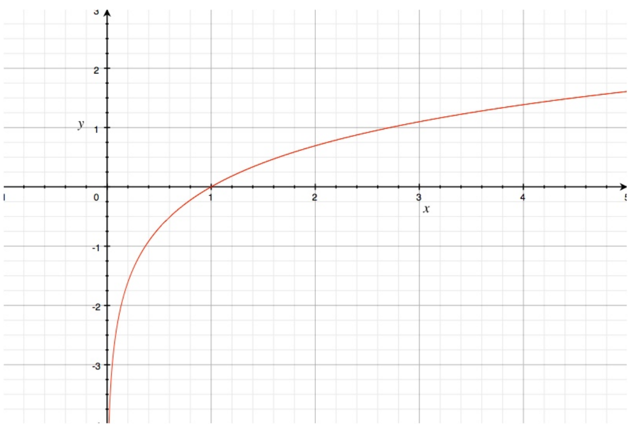
4) What can tou tell about the montonicity of g'?

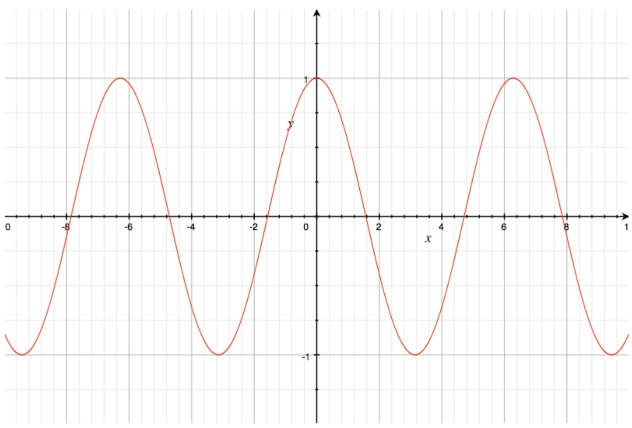
Definition. We say that a differentiable function is convex (or concave up) if its derivative is an increasing function. We say that a differentiable function is concave (or concave down) if its derivative is a decreasing function.

5) Just by looking at their graph, tell if the following functions are convex or concave: the exponential function, the square root function, the ln function and the cosinus function.









6) Compute the second-order derivative of all those functions. What is the link between the second-order derivative and the convexity or concavity of the function. Why does it make sense?

Theorem. *Let f be twice differentiable on some interval I. If $f" \geq 0$ on this interval, then f is convex. If $f" \leq 0$, then f is concave.*

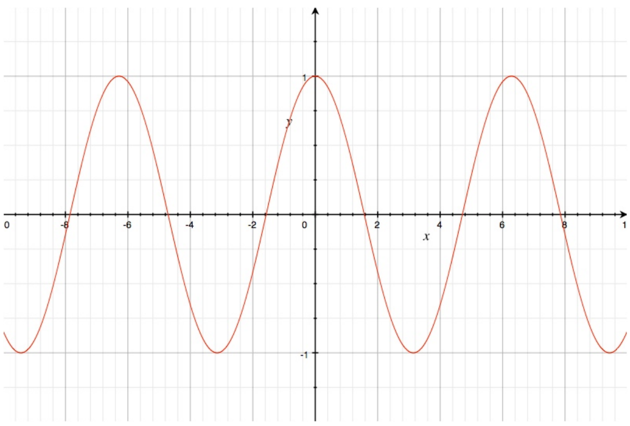
7) Are there intervals where the function cos is concave or convex? What happens to the second-order derivative of cos when the concavity of the function changes?

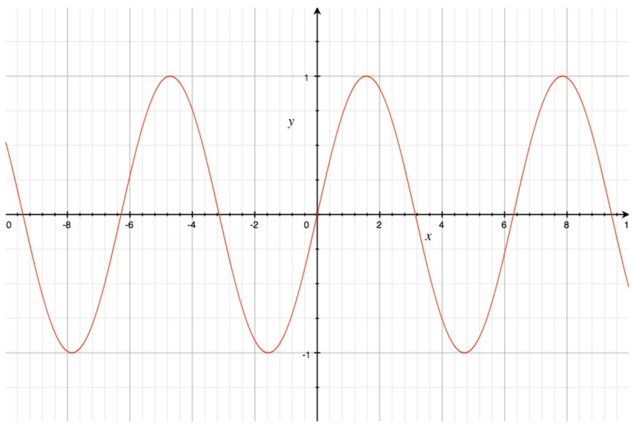
Definition. We say that c is an inflexion point of f is the monotonicity of f' changes at c.

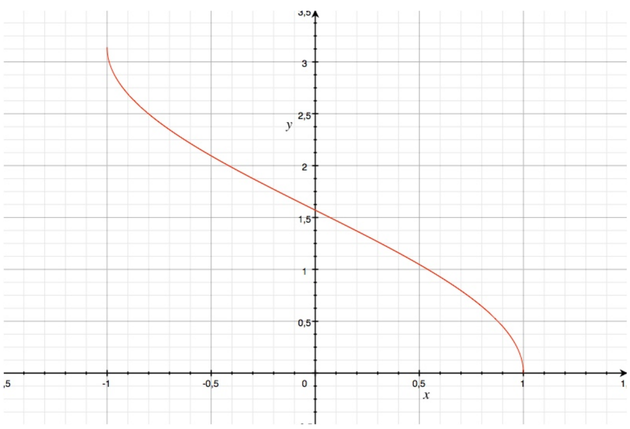
Complete the following theorem

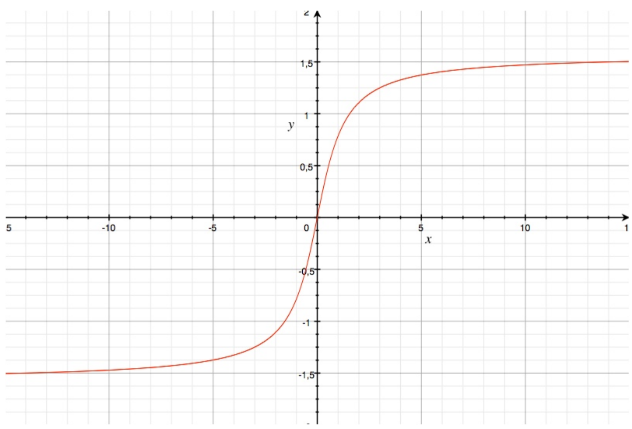
Theorem. *Let f be twice differentiable. If $f"$, then c is an inflexion point of f.*

8) On the following graphs of the functionns cos, sin, $\cos^{-1}$ and $\tan^{-1}$, mark the inflexion points and highline the function in yellow when the function is concave and in orange when the function is convex.









9) Complete the blanks in the following text:

"Assume that $f'(c) = 0$, then we ______ (do/don't) know whether $f(c)$ is a local extremum of f. Let us assume furthermore that $f''(c) > 0$, then we ______ (do/don't) know whether the sign of f' changes at c. This implies that $f(c)$ is in fact a ______ of f.

Theorem. *If f is twice differentiable on an interval containing c and $f'(c) = 0$. Then,*

if $f''(c) > 0$, then $f(c)$ is a local ______ of f

if $f''(c) < 0$, then $f(c)$ is a local ______ of f

Let $f(x) = (x-1)^2$, $g(x) = (x-1)^3$ and $h(x) = -(x-1)^4$.

10) Compute $f''(1)$, $g''(1)$ and $h''(1)$

Theorem. *If f is twice differentiable on an interval containing c and $f'(c) = 0$. Then,*

if $f''(c) > 0$, then $f(c)$ is a local ______ of f

if $f''(c) < 0$, then $f(c)$ is a local ______ of f

if $f''(c) = 0$, then