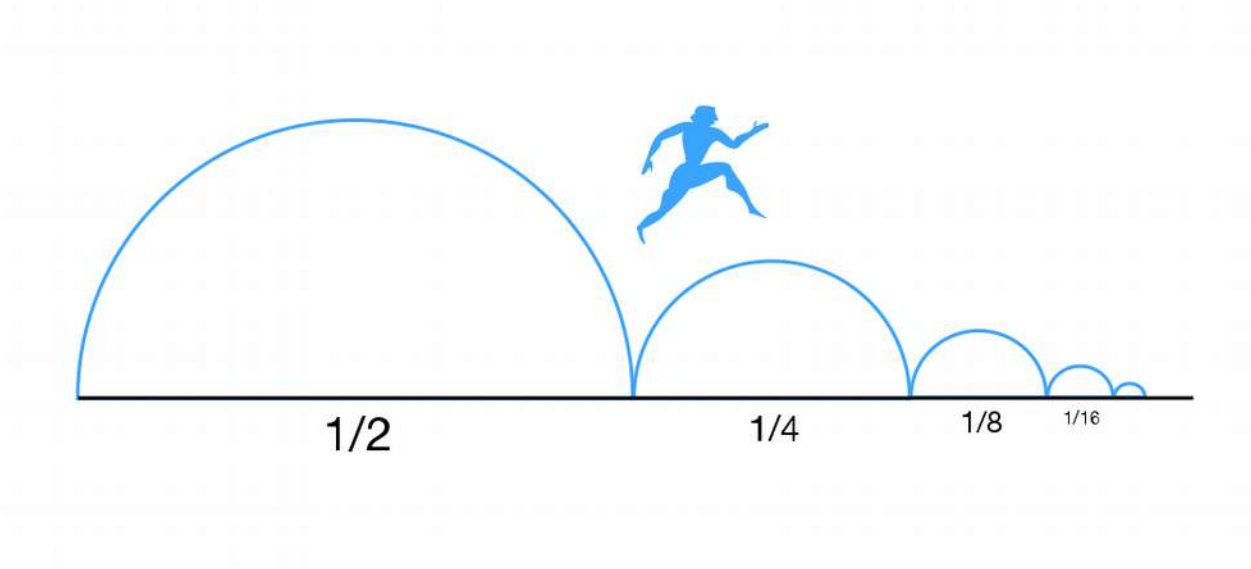
Exercise 1 Zenon's paradox

Zenon is (490-430 BC) was a Greek philosopher. One of his famous paradoxes (check wikipedia: Zenon's paradoxes) states that motion is impossible. Here is his proof: Suppose that Hercules has to go from one point A to one point B, which are separated by 1 mile. Before reaching point B, Hercules will have to walk half the distance (step 1). Once Hercules has reach the middle of A and B, he now has to walk half of the remaining distance (therefore, one quarter of the total distance) (step 2). But once Hercules has reach this point, he still has to walk half of the remaining distance (step 3) and so on... Finally, Hercules will never reach point B because it would take him an infinite amount of step to complete this task.



1. Why does Zenon conclude that motion is impossible ?

2. How would you solve this paradox ?

3. How many miles has Hercules walked during the first step ? During the second ? During the third ?

4. How many miles has Hercules walked during the *n*-th step ?

5. How far is Hercules from B after the first step ? After the second ? After the third ?

6. How far is Hercules from B after the *n*-th step ? Let us call this distance Z_n.

7. How does Z_n evolve when *n* gets larger and larger ?

Exercise 2 A second paradox

Let f:x $\rightarrow x^2$ and g:x $\rightarrow$ 1/x.

1. Compute g(10), g(100), g(1000), g(1,000,000).

2. What can you say about g(x) when x is very large ?

3. Does g(x)=0 have a solution ?

4. Compute g(0.1), g(0.01), g(0.001), g(0.000,001) ?

5. What can you say about g(x) when x is very small ?

6. On the next page is the graph of f on [0,4]. Draw the line between (1,1) and $(3,3^2)$. What is the slope of this line ? Draw the line between (1,1) and $(2,2^2)$. What is the slope of this line ?

7. On the next page is the graph of f on [1,2]. Draw the line between (1,1) and $(1.5, 1.5^2)$. What is the slope of this line ? Draw the line between (1,1) and $(1.1,1.1^2)$. What is the slope of this line ?

8. What is the slope of the line between (1,1) and $(1+x,(1+x)^2)$? Let's call df_x the value of this slope.

9. How do you think df_x evolves as x gets smaller and smaller ?

10. Write df_x as the product of a function with g(x). Do you see a paradox ?

11. How would you solve this paradox ?

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Introduction to limits